A new method to measure pulsed RF time domain waveforms with a sub-sampling system

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Abstract—This paper describes a new method that enables time domain pulsed RF measurements with any sub-sampling system. The method consists of replacing the FFT with a rectangular windowed short-time Fourier transform (STFT). The algorithm automatically extracts Fourier coefficients within the pulses and the system does not need any trigger signal or clocking circuit. This minimal software modification enhances the standard Large Signal Network Analyzer (LSNA), enabling pulsed measurements without any hardware modifications. Our algorithm has been tested with a wideband amplifier at 1.5 GHz.

Index Terms—Pulsed RF, time domain measurement, multi-harmonic, large signal network analyzer (LSNA).

I. INTRODUCTION

Some multi-harmonic large signal measurements on high power transistors have to be performed with pulsed RF signals in order to minimize dissipated power. A nonlinear network analyzer (NVNA) [1] performs pulsed measurements using averaging resulting from the narrow-band frequency domain approach [2]. Currently, measuring time-domain waveforms with a conventional LSNA [3] is not possible under pulsed RF conditions. A stroboscopic approach has been demonstrated in e.g. [4][5] but requires complex hardware modifications. In contrast, the work described in this paper presents a new method that enables pulsed RF measurements using an existing LSNA with no additional hardware requirements. The method requires substituting the FFT applied to raw data by a Short Time Fourier Transform (STFT). The method is applied at 1.5GHz to a wideband amplifier using a subsampling system.

VNA and NVNA setups perform RF pulsed measurements in frequency domain and can be calibrated in CW mode. During the pulsed RF measurements, only the power at the center frequency (power \( P_1 \)) is measured and RF pulsed \( P_{1p} \) is calculated as [6] :

\[
P_{1p} = P_1 \left( \frac{T}{\tau_p} \right)^2
\]

where \( \tau_p \) is the pulse width and \( T \) the pulse repetition rate. Such measurements are performed sequentially at \( f_0 \) and several harmonics when using an NVNA in order to extract the RF time domain waveform within the pulse [2].

Unfortunately, for a sampler based setup such as a LSNA, due to the compressive nature of the sub-sampling, aliasing prevents retrieving information in the frequency domain, illustrated on Fig. 1 using raw data for \( \tau_p = 10\mu s \) and \( T = 100\mu s \). The aliasing problem can be surmounted by sorting the ADC samples [4] [5]. Although this method does not reduce dynamic range, it requires accurate timing and triggering accompanied with a complex circuitry.

![Fig. 1. Raw data from a LSNA during 10\mu s/100\mu s (\tau_p/T) pulsed RF measurements. The time-domain data (left) is transformed with a FFT resulting in spectrum aliasing (right) which prevents standard frequency domain analysis. The LSNA configuration is detailed on Table I.](image)

An alternate approach, described theoretically in the next section, replaces standart frequency analysis with a time-frequency analysis assuming that the pulsed RF signal is a multi-harmonic CW. No hardware modification of the LSNA is required. Furthermore, we do not have to know the pulse duration or period as in the NVNA solution.

II. MATHEMATICAL BACKGROUND

In this part, \( x(t) \) denotes a signal acquired on a LSNA’s ADC. Let us consider a dictionary as :

\[
D = \{ \psi_k \}_{k \in \Gamma}
\]

As long as the dictionary is an orthonormal basis, any finite energy signal \( x(t) \) can be represented by its inner-product coefficients :

\[
\langle x, \psi_k \rangle = \int_{-\infty}^{+\infty} x(t) \cdot \overline{\psi_k}(t) \, dt
\]

where \( \overline{\psi_k} \) is the complex conjugate of \( \psi_k \).

We can calculate a linear approximation that projects the signal in a space of lowest possible dimension. The
signal energy is concentrated over a fewer set of vectors:

\[ x(t) \approx \sum_{k \in \Lambda} \langle x, \psi_k \rangle \psi_k \tag{4} \]

CW time-domain measurements with a LSNA are based on Fourier analysis. In this case, the dictionary used to project a signal (2) is defined as:

\[ D = \{ \psi_f(t) = e^{j2\pi f t} \} \tag{5} \]

and therefore 3-4 became:

\[ x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \tag{6} \]

\[ X(f) = \langle x, \psi_f \rangle = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \tag{7} \]

and equation (4) becomes:

\[ x(t) \approx \sum_k X(k, f_0) e^{j2\pi k f_0 t} \tag{8} \]

where \( f_0 \) is the fundamental frequency and the set of values for \( k \) defines the frequency grid for harmonic analysis as we define it in an harmonic balance simulation or a LSNA measurement [7]. The Fourier transform is valid only for linear time-invariant systems. Thus, FFT based LSNA measurements are well suited only for CW measurements.

For nonstationary signals, Gabor [8] introduced a time/frequency description thanks to a local Fourier analysis with sliding Gaussian window. Stationarity is assumed within the window. The continuous-time Short-time Fourier transform is defined as:

\[ D = \{ \psi_{f,\tau}(t) = w(t - \tau), e^{j2\pi f t} \} \tag{9} \]

and

\[ X(f, \tau) = \langle x, \psi_{f,\tau} \rangle = \int_{-\infty}^{\infty} x(t)w(t - \tau) e^{-j2\pi f t} dt \tag{10} \]

Usually, the window function \( w(t) \) is real and gaussian in order to generate a bandlimited atom in the time-frequency plane. In this work, we apply a rectangular window that is well suited for multi-harmonic CW signals:

\[ w(t) = \Pi(t) = \begin{cases} 0, & \text{if } |t| > \frac{1}{2} \\ 1, & \text{if } |t| < \frac{1}{2} \\ \frac{1}{2}, & \text{if } |t| = \frac{1}{2} \end{cases} \tag{11} \]

The idea is to perform an exact sliding FFT on the signal \( x(t) \). This signal is assumed to be multi-harmonic as \( x(t) = \sum_k A_k e^{j2\pi k f_0 t} \) where \( f_0 \) is the fundamental frequency. The rectangular window width is equal to the period of the fundamental frequency. It ensures no aliasing in the calculation of \( X(f, \tau) \) thanks to the location of the zeros in the window’s frequency response (sinc function). Then the dictionary can be written as:

\[ \psi_{k,\tau}(t) = P_k \psi_k(t - \tau) \tag{12} \]

with

\[ P_k = f_0 e^{j2\pi k f_0 \tau} \tag{13} \]

and

\[ \psi_k(t) = \Pi(f_0 t) e^{j2\pi k f_0 t} \tag{14} \]

\( P_k \) is a normalization factor that ensures:

\[ \int_{-\infty}^{\infty} \psi_{k,\tau}(t) dt = 1 \tag{15} \]

and a phase self consistance of the inner-product for every value of \( \tau \).

According to equation (10), the Fourier coefficients are:

\[ X(k, f_0, \tau) = \mathcal{P}_k x(t) \star \bar{\psi}_k(t) \tag{16} \]

When \( x(t) \) is periodic, equation (16) becomes a circular convolution, and the STFT is better calculated in the frequency domain according to:

\[ X(k, f_0, \tau) = \mathcal{F}^{-1}\{X(f) \cdot \overline{\Psi}_k(f)\} \tag{17} \]

where \( X(f) \) and \( \Psi_k(f) \) are the Fourier transform of the signal \( x(t) \) and the window \( \psi_k(t) \), respectively. The operator \( \mathcal{F}^{-1}\{\cdot\} \) denotes inverse Fourier transform. In our computer implementation, \( X(f) \) is calculated at every measurement acquisition, but \( \Psi_k(f) \) is calculated for all values of the frequency parameter \( k \) just once, before measurements, during the setup initialization.

III. LSNA MEASUREMENTS EXAMPLE

The new algorithm, based on equation (17), has been embedded in the LSNA software to replace the standard FFT. RF measurements were performed on an ultra wideband amplifier [9] in order to measure harmonics at the output of the DUT. The parameters used on the LSNA are displayed in Table I. The amplifier is driven by a 100\( \mu \)s pulse period and several duty cycle values from 10\% to 100\% (CW). This amplifier was previously measured on a commercial NVNA setup and exhibits the same time domain waveforms for all those duty cycle values. For this kind of characterization, a NVNA needs the duty cycle value in order to correct the power as explained in equation (1), but the LSNA can adaptively sample the pulsed RF and extract automatically the correct RF magnitudes and phases within the pulse irrespective of its width. The minimal measurable pulse width has to be larger than the intermediate fundamental frequency period (8\( \mu \)s in this example).

In our case, ADCs are not triggered and the pulse time-location is totally arbitrary at every measurement.
TABLE I
LSNA CONFIGURATION USED IN THIS PAPER

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF frequency</td>
<td>1.5 GHz</td>
</tr>
<tr>
<td>Number of harmonics</td>
<td>4</td>
</tr>
<tr>
<td>FracN frequency</td>
<td>19.998333 MHz</td>
</tr>
<tr>
<td>ADC frequency</td>
<td>20 MHz</td>
</tr>
<tr>
<td>ADC number of points</td>
<td>20 000</td>
</tr>
<tr>
<td>ADC FFT bins of interest</td>
<td>126, 251, 376 and 501</td>
</tr>
<tr>
<td>IF fundamental</td>
<td>125 kHz</td>
</tr>
<tr>
<td>Analyzing window width</td>
<td>8 µs</td>
</tr>
</tbody>
</table>

A gate profile is then defined thanks to a threshold level on the $|X(k,f_0,t)|$ trace in order to extract Fourier coefficients into the pulses only. Figure 2 illustrates a raw data aquisition on a LSNA channel and its STFT. Only the coefficients located within the gate are taken into account. In this example, the gate width is narrower than 2µs.

This measurement method make possible to extract RF harmonic information at any time within the pulse such as the envelope transient analysis. It is not limited to a periodic frame for the RF modulation and may be usefull for investigations with bursts of pulses conditions.

RF measurements performed with the LSNA are displayed in figure 3 as they appear to the user. Those measurements are exported in an Agilent ADS CITIfile as CW measurements and a pulse profile could be exported too as an envelope transient simulation data-set.

IV. CONCLUSION

In this work, we discussed the theoretical foundations and practical application of the pulsed multi-harmonic time domain waveforms approach for sub-sampling measurements. This new method, intended to replace the FFT applied on raw data, is adaptive and consistent with both CW and pulsed RF signals. Only the FFT, has to be modified in the standard LSNA software, and it could be implemented within other sub-sampling downconverters systems.

REFERENCES