

TRL algorithm to de-embed a RF test fixture

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1 TRL

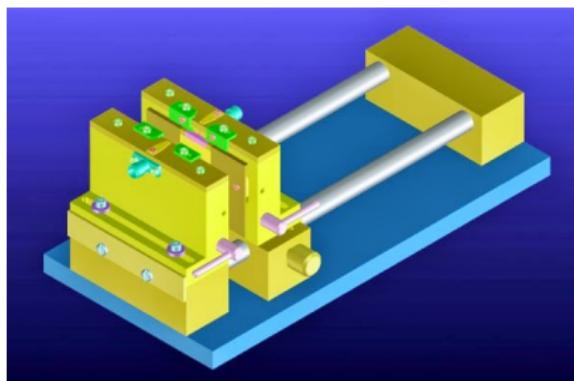
- Standards
- THRU and LINE Measurements
- T_{OUT} parameters : $\left(\frac{T_{12}}{T_{11}}\right)$ and $\left(\frac{T_{21}}{T_{22}}\right)$
- \bar{T}_{IN} parameters : $\left(\frac{\bar{T}_{12}}{\bar{T}_{11}}\right)$ and $\left(\frac{\bar{T}_{21}}{\bar{T}_{22}}\right)$
- The THRU equality : $\left(\frac{T_{11}}{\bar{T}_{11}}\right)$ and $\left(\frac{T_{21}}{\bar{T}_{22}}\right)$
- The REFLECT equality : Extracting $\left(\frac{\bar{T}_{21}}{\bar{T}_{11}}\right)$

2 Reciprocity

3 Scilab Code

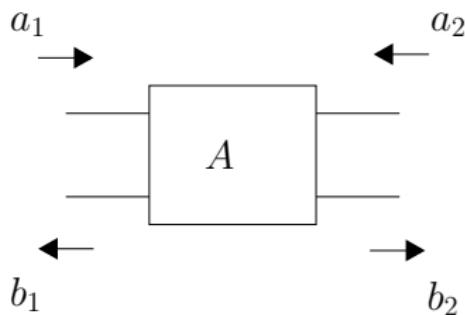
- Presentation
- Example
- Insights

Motivations for this talk



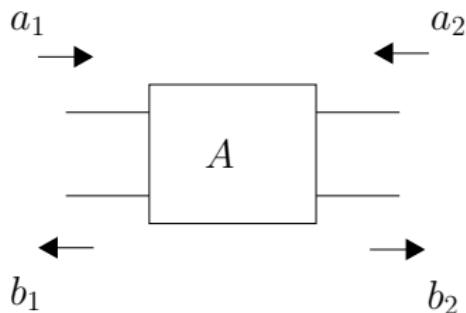
- De-embedding of a test-fixture measured in the coaxial reference planes ;
- Perform a TRL calibration when the VNA provides some ill conditioned solutions ("Reflect" checking not correct) ;
- Offer a open, complete and ready-to-use source code for educational purpose.

Some definitions : [S] parameters



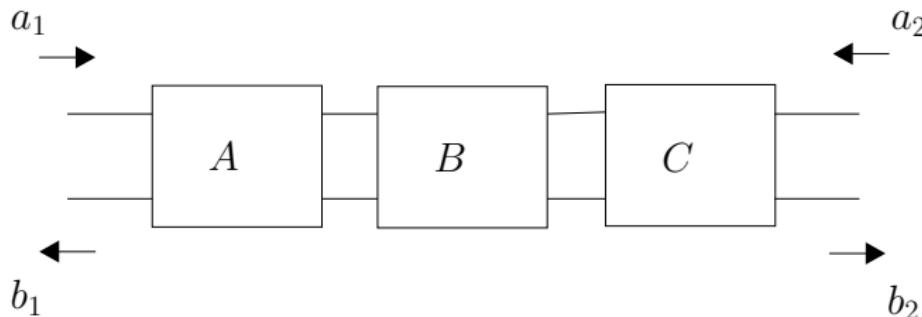
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad (1)$$

Some definitions : [T] parameters



$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \begin{pmatrix} b_2 \\ a_2 \end{pmatrix} \quad (2)$$

Some definitions : [T] parameters



$$[T_{Total}] = [T_A] \cdot [T_B] \cdot [T_C] \quad (3)$$

Conversions between [S] and [T]

- [S] to [T]

$$[T] = \begin{bmatrix} -\frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & \frac{S_{21} \cdot S_{12} - S_{11} \cdot S_{22}}{S_{21}} \end{bmatrix} \quad (4)$$

- [T] to [S]

$$[S] = \begin{bmatrix} -\frac{T_{21}}{T_{11}} & \frac{T_{11} \cdot T_{22} - T_{12} \cdot T_{21}}{T_{11}} \\ \frac{1}{T_{11}} & -\frac{T_{12}}{T_{11}} \end{bmatrix} \quad (5)$$

Standards for the TRL algorithm

- THRU : totally known

$$\left[T_{THRU}^{Std} \right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (6)$$

- REFLECT : unknown

$$\text{Sgn} \left(\Re \left\{ \Gamma_{REFLECT}^{Std} \right\} \right) = \pm 1 \quad (7)$$

- LINE : partially known

$$\left[T_{LINE}^{Std} \right] = \begin{bmatrix} e^{-\gamma \cdot I} & 0 \\ 0 & e^{+\gamma \cdot I} \end{bmatrix} \quad (8)$$

Measuring the THRU

$$\left[T_{THRU}^{Meas} \right] = [T_{IN}] \cdot \left[T_{THRU}^{Std} \right] \cdot [T_{OUT}] \quad (9)$$

$$[T_{IN}]^{-1} \cdot [T_{THRU}^{Meas}] = [T_{THRU}^{Std}] \cdot [T_{OUT}] \quad (10)$$

$$[T_{IN}]^{-1} \cdot [T_{THRU}^{Meas}] = [T_{OUT}] \quad (11)$$

$$[T_{IN}]^{-1} = [T_{IN}]$$

$$[T_{IN}] \cdot [T_{THRU}^{Meas}] = [T_{OUT}] \quad (12)$$

Measuring the LINE

$$\left[T_{LINE}^{Meas} \right] = [T_{IN}] \cdot \left[T_{LINE}^{Std} \right] \cdot [T_{OUT}] \quad (13)$$

$$[T_{IN}]^{-1} \cdot [T_{LINE}^{Meas}] = [T_{LINE}^{Std}] \cdot [T_{OUT}] \quad (14)$$

$$[T_{IN}] \cdot [T_{LINE}^{Meas}] = \begin{bmatrix} e^{-\gamma \cdot I} & 0 \\ 0 & e^{+\gamma \cdot I} \end{bmatrix} \cdot [T_{OUT}] \quad (15)$$

OUTPUT : Defining the [M] matrix

Equation (15) is :

$$\begin{bmatrix} \bar{T}_{11} & \bar{T}_{12} \\ \bar{T}_{21} & \bar{T}_{22} \end{bmatrix} \cdot \begin{bmatrix} T_{LINE}^{Meas} \end{bmatrix} = \begin{bmatrix} T_{11} \cdot e^{-\gamma \cdot I} & T_{12} \cdot e^{-\gamma \cdot I} \\ T_{21} \cdot e^{+\gamma \cdot I} & T_{22} \cdot e^{+\gamma \cdot I} \end{bmatrix} \quad (16)$$

(12) in (16) give :

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \left[T_{THRU}^{Meas} \right]^{-1} \cdot \left[T_{LINE}^{Meas} \right] = \begin{bmatrix} T_{11} \cdot e^{-\gamma \cdot I} & T_{12} \cdot e^{-\gamma \cdot I} \\ T_{21} \cdot e^{+\gamma \cdot I} & T_{22} \cdot e^{+\gamma \cdot I} \end{bmatrix} \quad (17)$$

or

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot [M] = \begin{bmatrix} T_{11} \cdot e^{-\gamma \cdot I} & T_{12} \cdot e^{-\gamma \cdot I} \\ T_{21} \cdot e^{+\gamma \cdot I} & T_{22} \cdot e^{+\gamma \cdot I} \end{bmatrix} \quad (18)$$

with

$$[M] = \begin{bmatrix} T_{THRU}^{Meas} \end{bmatrix}^{-1} \cdot \begin{bmatrix} T_{LINE}^{Meas} \end{bmatrix}$$

OUTPUT : TRL Equations

Equations given by (18) are :

$$T_{11} \cdot M_{11} + T_{12} \cdot M_{21} = T_{11} \cdot e^{-\gamma \cdot l} \quad (19)$$

$$T_{11} \cdot M_{12} + T_{12} \cdot M_{22} = T_{12} \cdot e^{-\gamma \cdot l} \quad (20)$$

$$T_{21} \cdot M_{11} + T_{22} \cdot M_{21} = T_{21} \cdot e^{+\gamma \cdot l} \quad (21)$$

$$T_{21} \cdot M_{12} + T_{22} \cdot M_{22} = T_{22} \cdot e^{+\gamma \cdot l} \quad (22)$$

OUTPUT : Solving $\left(\frac{T_{12}}{T_{11}}\right)$

(20) gives :

$$e^{-\gamma \cdot I} = \left(\frac{T_{11}}{T_{12}} \right) \cdot M_{12} + M_{22} \quad (23)$$

(23) in (19) gives :

$$T_{11} \cdot M_{11} + T_{12} \cdot M_{21} = T_{11} \cdot \left[\left(\frac{T_{11}}{T_{12}} \right) \cdot M_{12} + M_{22} \right] \quad (24)$$

$$M_{11} + \left(\frac{T_{12}}{T_{11}} \right) \cdot M_{21} = \left(\frac{T_{11}}{T_{12}} \right) \cdot M_{12} + M_{22} \quad (25)$$

$$\left(\frac{T_{12}}{T_{11}} \right)^2 \cdot M_{21} + \left(\frac{T_{12}}{T_{11}} \right) \cdot (M_{11} - M_{22}) - M_{12} = 0 \quad (26)$$

OUTPUT : Solving $\left(\frac{T_{22}}{T_{21}}\right)$

(21) gives :

$$e^{+\gamma \cdot I} = M_{11} + \left(\frac{T_{22}}{T_{21}}\right) \cdot M_{21} \quad (27)$$

(27) in (22) gives :

$$T_{21} \cdot M_{12} + T_{22} \cdot M_{22} = T_{22} \cdot \left[\left(\frac{T_{22}}{T_{21}}\right) \cdot M_{21} + M_{11} \right] \quad (28)$$

$$M_{22} + \left(\frac{T_{21}}{T_{22}}\right) \cdot M_{12} = \left(\frac{T_{22}}{T_{21}}\right) \cdot M_{21} + M_{11} \quad (29)$$

$$\left(\frac{T_{22}}{T_{21}}\right)^2 \cdot M_{21} + \left(\frac{T_{22}}{T_{21}}\right) \cdot (M_{11} - M_{22}) - M_{12} = 0 \quad (30)$$

OUTPUT : $\left(\frac{T_{12}}{T_{11}}\right)$ and $\left(\frac{T_{22}}{T_{21}}\right)$

$$X^2 \cdot M_{21} + X \cdot [M_{11} - M_{22}] - M_{12} \quad (31)$$

This polynom has 2 solutions : $\left(\frac{T_{12}}{T_{11}}\right)$ and $\left(\frac{T_{22}}{T_{21}}\right)$

Usually, $\left|\frac{T_{12}}{T_{11}}\right| < \left|\frac{T_{22}}{T_{21}}\right|$

If we consider the following polynom :

$$X^2 \cdot M_{12} + X \cdot [M_{22} - M_{11}] - M_{21} \quad (32)$$

Then the 2 solutions are $\left(\frac{T_{11}}{T_{12}}\right)$ and $\left(\frac{T_{21}}{T_{22}}\right)$

INPUT : Defining the [N] matrix

Equation (15) is :

$$\begin{bmatrix} \bar{T}_{11} & \bar{T}_{12} \\ \bar{T}_{21} & \bar{T}_{22} \end{bmatrix} \cdot \begin{bmatrix} T_{LINE}^{Meas} \end{bmatrix} = \begin{bmatrix} e^{-\gamma \cdot I} & 0 \\ 0 & e^{+\gamma \cdot I} \end{bmatrix} \cdot \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad (33)$$

(12) in (33) gives :

$$\begin{bmatrix} \bar{T}_{11} & \bar{T}_{12} \\ \bar{T}_{21} & \bar{T}_{22} \end{bmatrix} \cdot \begin{bmatrix} T_{LINE}^{Meas} \end{bmatrix} = \begin{bmatrix} e^{-\gamma \cdot I} & 0 \\ 0 & e^{+\gamma \cdot I} \end{bmatrix} \cdot \begin{bmatrix} \bar{T}_{11} & \bar{T}_{12} \\ \bar{T}_{21} & \bar{T}_{22} \end{bmatrix} \cdot \begin{bmatrix} T_{THRU}^{Meas} \end{bmatrix} \quad (34)$$

or

$$\begin{bmatrix} \bar{T}_{11} & \bar{T}_{12} \\ \bar{T}_{21} & \bar{T}_{22} \end{bmatrix} \cdot [N] = \begin{bmatrix} \bar{T}_{11} \cdot e^{-\gamma \cdot I} & \bar{T}_{12} \cdot e^{-\gamma \cdot I} \\ \bar{T}_{21} \cdot e^{+\gamma \cdot I} & \bar{T}_{22} \cdot e^{+\gamma \cdot I} \end{bmatrix} \quad (35)$$

with

$$[N] = \left[T_{LINE}^{Meas} \right] \cdot \left[T_{THRU}^{Meas} \right]^{-1}$$

INPUT : $\left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right)$ and $\left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right)$

Equation (35) is similar to (18). Thus we can consider :

$$X^2 \cdot N_{21} + X \cdot [N_{11} - N_{22}] - N_{12} \quad (36)$$

This polynom has 2 solutions : $\left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right)$ and $\left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right)$

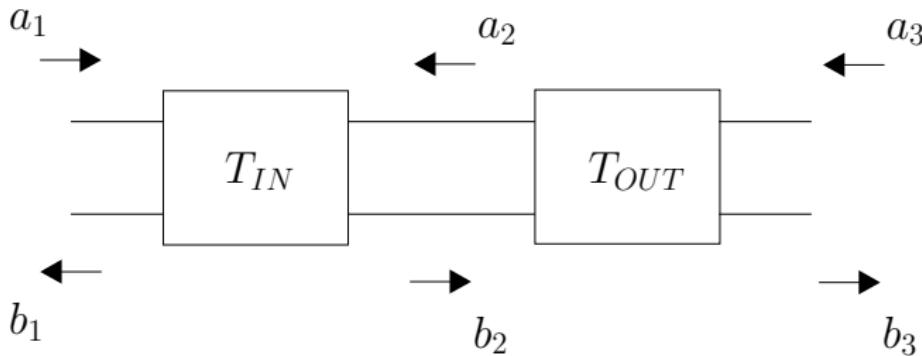
Usually, $\left|\frac{\overline{T}_{12}}{\overline{T}_{11}}\right| < \left|\frac{\overline{T}_{22}}{\overline{T}_{21}}\right|$

If we consider the following polynom :

$$X^2 \cdot N_{12} + X \cdot [N_{22} - N_{11}] - N_{21} \quad (37)$$

Then the 2 solutions are $\left(\frac{\overline{T}_{11}}{\overline{T}_{12}}\right)$ and $\left(\frac{\overline{T}_{21}}{\overline{T}_{22}}\right)$

The THRU equality



$$\begin{pmatrix} b_2 \\ a_2 \end{pmatrix} = \begin{bmatrix} \overline{T}_{11} & \overline{T}_{12} \\ \overline{T}_{21} & \overline{T}_{22} \end{bmatrix} \cdot \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

$$\begin{pmatrix} b_2 \\ a_2 \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \begin{pmatrix} b_3 \\ a_3 \end{pmatrix}$$

Forward mode : b_2 equality to extract $\left(\frac{T_{11}}{\bar{T}_{11}} \right)$

The b_2 equality leads us to :

$$\bar{T}_{11} \cdot a_1 + \bar{T}_{12} \cdot b_1 = T_{11} \cdot b_3 + T_{12} \cdot a_3 \quad (38)$$

And by definition, about the THRU measurement, we know :

$$S_{21}^{Meas} = \left. \frac{b_3}{a_1} \right|_{a_3=0} \text{ and } S_{11}^{Meas} = \left. \frac{b_1}{a_1} \right|_{a_3=0}$$

Thus,

$$a_1 \cdot \left(\bar{T}_{11} + \bar{T}_{12} \cdot \frac{b_1}{a_1} \right) = T_{11} \cdot b_3 \quad (39)$$

$$\bar{T}_{11} \cdot \left(1 + \left(\frac{\bar{T}_{12}}{\bar{T}_{11}} \right) \cdot S_{11}^{Meas} \right) = T_{11} \cdot S_{21}^{Meas} \quad (40)$$

$$\left(\frac{T_{11}}{\bar{T}_{11}} \right) = \frac{\left(1 + \left(\frac{\bar{T}_{12}}{\bar{T}_{11}} \right) \cdot S_{11}^{Meas} \right)}{S_{21}^{Meas}} \quad (41)$$

Reverse mode : a_2 equality to extract $\left(\frac{T_{21}}{\bar{T}_{21}} \right)$

The a_1 equality leads us to :

$$\bar{T}_{21} \cdot a_1 + \bar{T}_{22} \cdot b_1 = T_{21} \cdot b_3 + T_{22} \cdot a_3 \quad (42)$$

For the THRU measurement, we know :

$$S_{12}^{Meas} = \left. \frac{b_1}{a_3} \right|_{a_1=0} \text{ and } S_{22}^{Meas} = \left. \frac{b_3}{a_3} \right|_{a_1=0}$$

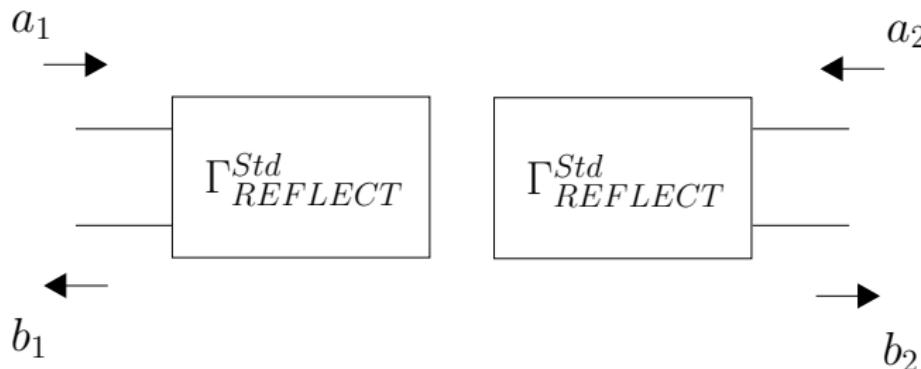
Thus,

$$b_1 \cdot \bar{T}_{22} = T_{21} \cdot b_3 + T_{21} \cdot a_3 \quad (43)$$

leads us to :

$$\left(\frac{T_{21}}{\bar{T}_{22}} \right) = \frac{S_{12}^{Meas}}{S_{22}^{Meas} + \left(\frac{T_{22}}{\bar{T}_{21}} \right)} \quad (44)$$

REFLECT Measurement



$$\Gamma_{REFLECT}^{Std} = \frac{b_1}{a_1} = \frac{\bar{T}_{21} + \bar{T}_{22} \cdot S_{11}^{Meas}}{\bar{T}_{11} + \bar{T}_{12} \cdot S_{11}^{Meas}} \quad (45)$$

$$\Gamma_{REFLECT}^{Std} = \frac{b_2}{a_2} = \frac{T_{12} + T_{11} \cdot S_{22}^{Meas}}{T_{22} + T_{21} \cdot S_{22}^{Meas}} \quad (46)$$

REFLECT Equality

$$\frac{\overline{T}_{21} + \overline{T}_{22} \cdot S_{11}^{Meas}}{\overline{T}_{11} + \overline{T}_{12} \cdot S_{11}^{Meas}} = \frac{T_{12} + T_{11} \cdot S_{22}^{Meas}}{T_{22} + T_{21} \cdot S_{22}^{Meas}} \quad (47)$$

$$\frac{\overline{T}_{21}}{\overline{T}_{11}} \cdot \left(\frac{1 + \left(\frac{\overline{T}_{22}}{\overline{T}_{21}} \right) \cdot S_{11}^{Meas}}{1 + \left(\frac{\overline{T}_{12}}{\overline{T}_{11}} \right) \cdot S_{11}^{Meas}} \right) = \frac{T_{11}}{T_{21}} \cdot \left(\frac{S_{22}^{Meas} + \left(\frac{T_{12}}{T_{11}} \right)}{S_{22}^{Meas} + \left(\frac{T_{22}}{T_{21}} \right)} \right) \quad (48)$$

$$(\overline{T}_{21})^2 \cdot \left(\frac{T_{21}}{\overline{T}_{22}} \right) \cdot \left(\frac{\overline{T}_{22}}{\overline{T}_{21}} \right) = (\overline{T}_{11})^2 \cdot \left(\frac{T_{11}}{\overline{T}_{11}} \right) \cdot \frac{\left(\frac{S_{22}^{Meas} + \left(\frac{T_{12}}{T_{11}} \right)}{S_{22}^{Meas} + \left(\frac{T_{22}}{T_{21}} \right)} \right)}{\left(\frac{1 + \left(\frac{\overline{T}_{22}}{\overline{T}_{21}} \right) \cdot S_{11}^{Meas}}{1 + \left(\frac{\overline{T}_{12}}{\overline{T}_{11}} \right) \cdot S_{11}^{Meas}} \right)} \quad (49)$$

REFLECT Equality

$$\left(\frac{\bar{T}_{21}}{\bar{T}_{11}}\right) = \pm \sqrt{\frac{\left(\frac{T_{11}}{\bar{T}_{11}}\right) \cdot \left(\frac{S_{22}^{Meas} + \left(\frac{T_{12}}{\bar{T}_{11}}\right)}{S_{22}^{Meas} + \left(\frac{T_{22}}{\bar{T}_{21}}\right)}\right)}{\left(\frac{T_{21}}{\bar{T}_{22}}\right) \cdot \left(\frac{\bar{T}_{22}}{\bar{T}_{21}}\right) \cdot \left(\frac{1 + \left(\frac{\bar{T}_{22}}{\bar{T}_{21}}\right) \cdot S_{11}^{Meas}}{1 + \left(\frac{\bar{T}_{12}}{\bar{T}_{11}}\right) \cdot S_{11}^{Meas}}\right)}} \quad (50)$$

There are 2 solutions. We select the good one thanks to the knowledge of $\text{Sgn}(\Re\{\Gamma_{REFLECT}^{Std}\}) = \pm 1$ in (45) :

$$\Gamma_{REFLECT}^{Std} = \left(\frac{\bar{T}_{21}}{\bar{T}_{11}}\right) \cdot \left(\frac{1 + \left(\frac{\bar{T}_{22}}{\bar{T}_{21}}\right) \cdot S_{11}^{Meas}}{1 + \left(\frac{\bar{T}_{12}}{\bar{T}_{11}}\right) \cdot S_{11}^{Meas}}\right) \quad (51)$$

TRL Completed

The TRL algorithm is completed. We got 7 parameters from :

- The $[M]$ matrix : $\left(\frac{T_{12}}{T_{11}}\right)$ and $\left(\frac{T_{22}}{T_{21}}\right)$ from (31) ;
- The $[N]$ matrix : $\left(\frac{\bar{T}_{12}}{\bar{T}_{11}}\right)$ and $\left(\frac{\bar{T}_{22}}{\bar{T}_{21}}\right)$ from (36) ;
- The THRU equality : $\left(\frac{T_{11}}{\bar{T}_{11}}\right)$ from (41) and $\left(\frac{T_{21}}{\bar{T}_{22}}\right)$ from (44) ;
- The REFLECT equality : $\left(\frac{\bar{T}_{21}}{\bar{T}_{11}}\right)$ from (50) ;

It is sufficient for $[S]$ parameters de-embedding but not for power measurement.

We need to normalize correctly the system of equation (finding the absolute value of \bar{T}_{11}).

For that purpose we will consider a reciprocity assumption :

$$S_{21}^{IN} = S_{12}^{IN}.$$

Reciprocity assumption

We should have :

$$\begin{bmatrix} \overline{T}_{IN} \end{bmatrix} = \begin{bmatrix} \frac{S_{21} \cdot S_{12} - S_{11} \cdot S_{22}}{S_{12}} & \frac{S_{22}}{S_{12}} \\ -\frac{S_{12}}{S_{11}} & \frac{1}{S_{12}} \end{bmatrix} \quad (52)$$

Thus the reciprocity assumption ($S_{21} = S_{12}$) leads to :

$$\overline{T}_{11} \cdot \overline{T}_{22} - \overline{T}_{12} \cdot \overline{T}_{21} = 1 \quad (53)$$

Reciprocity assumption

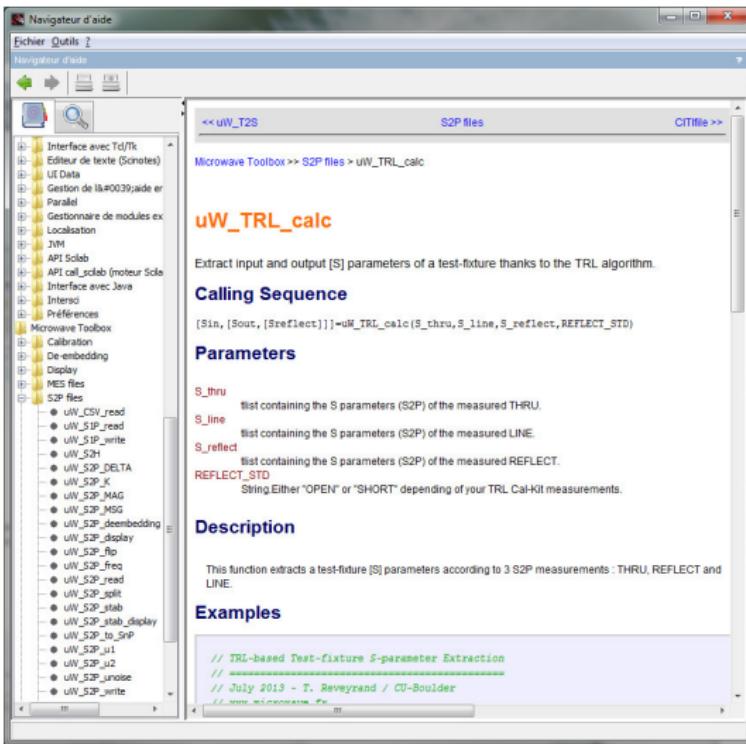
We can obtain from TRL the complete $[\bar{T}_{IN}]$ and $[\bar{T}_{OUT}]$ matrix from an arbitrary value of \bar{T}_{11} . Those matrix has to be multiplied by K in order to fullfill equation (53) such as :

$$K^2 = \frac{1}{\bar{T}_{11} \cdot \bar{T}_{22} - \bar{T}_{12} \cdot \bar{T}_{21}} \quad (54)$$

$$K = \pm \sqrt{\frac{1}{\bar{T}_{11} \cdot \bar{T}_{22} - \bar{T}_{12} \cdot \bar{T}_{21}}} \quad (55)$$

There are 2 solutions. The good one is selected such as the extrapolated phase of S_{21} on DC is as close as possible of zero.

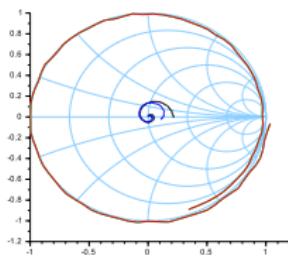
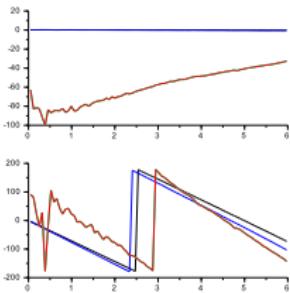
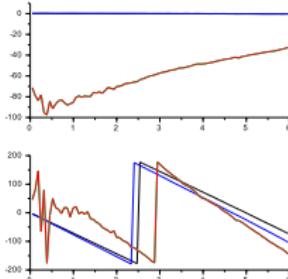
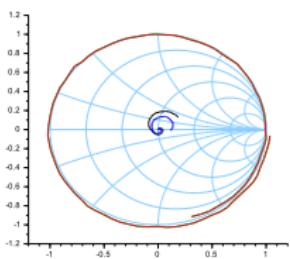
This code is now available in Scilab



<http://www.microwave.fr/uW.html>

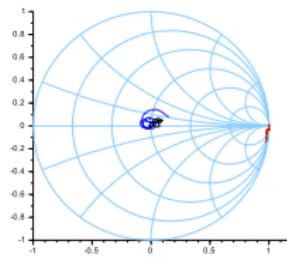
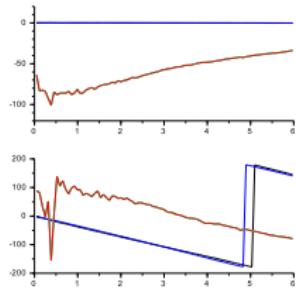
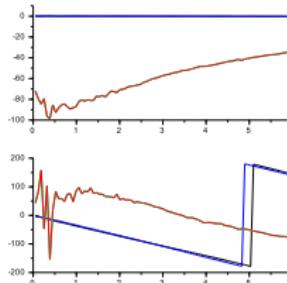
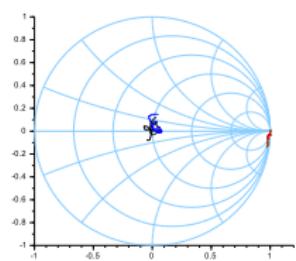
Example with an OPEN reflect

S2P Measurements : Thru (black), Line (blue) and Open (red).



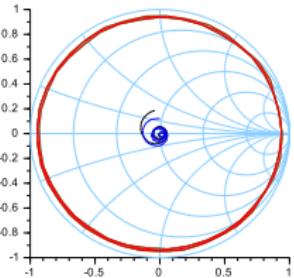
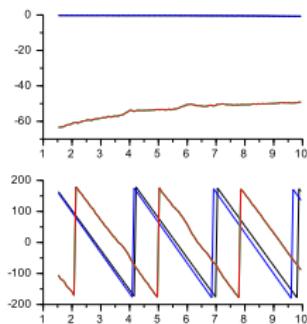
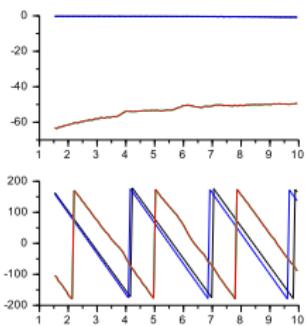
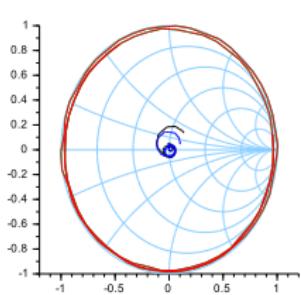
Example with an OPEN reflect

S2P Extracted : Port 1 (black), Port 2 (blue) and De-embedded open (red).



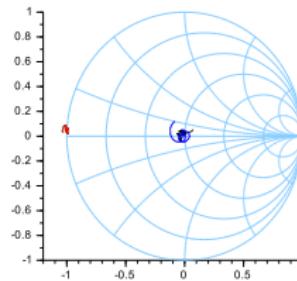
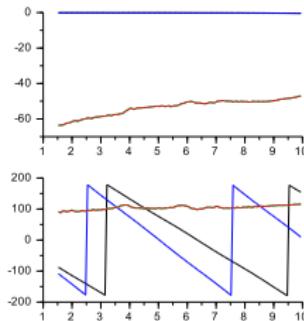
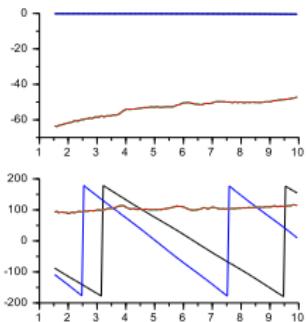
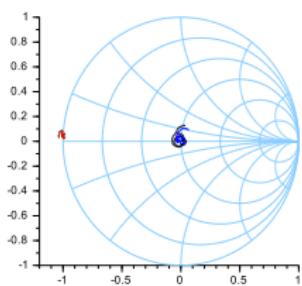
Example with an SHORT reflect

S2P Measurements : Thru (black), Line (blue) and Short (red).



Example with an SHORT reflect

S2P Extracted : Port 1 (black), Port 2 (blue) and De-embedded short (red).



Source Code : uW_TRL_calc.sci

```
function varargout=uW_TRL_calc(S_thru,S_line,S_reflect,REFLECT_STD)

T_THRU=uW_S2Z(S_thru);
T_LINE=uW_S2Z(S_line);

M11=[ ];M12=[ ];M21=[ ];M22=[ ];
N11=[ ];N12=[ ];N21=[ ];N22=[ ];

T_IN=list();
T_OUT=list();

K=[ ];

for i=1:size(T_THRU.frequency,1),
    T1=[T_THRU.T11(i),T_THRU.T12(i);T_THRU.T21(i),T_THRU.T22(i)];
    T2=[T_LINE.T11(i),T_LINE.T12(i);T_LINE.T21(i),T_LINE.T22(i)];
    M=inv(T1)*T2;
    N=T2*inv(T1);

    //Equation TRL_M...
    delta=(M(1,1)-M(2,2))^2-4*M(2,1)*(-M(1,2));
    X1=((M(2,2)-M(1,1))+sqrt(delta))/(2*M(2,1));
    X2=((M(2,2)-M(1,1))-sqrt(delta))/(2*M(2,1));
    Sol=[X1,X2];
    C1=Sol(find(abs(Sol)==max(abs(Sol)))); //OUTPUT - T22/T21
    C2=Sol(find(abs(Sol)==min(abs(Sol)))); //OUTPUT - T12/T11;

    //Equation TRL_N
    delta=(N(1,1)-N(2,2))^2-4*N(2,1)*(-N(1,2));
    X1=((N(2,2)-N(1,1))+sqrt(delta))/(2*N(2,1));
    X2=((N(2,2)-N(1,1))-sqrt(delta))/(2*N(2,1));
    Sol=[X1,X2];
    C3=Sol(find(abs(Sol)==max(abs(Sol)))); //INPUT - D/C
    C4=Sol(find(abs(Sol)==min(abs(Sol)))); //INPUT - B/A;

    //Equation THRU FORWARD
    C5=(1+C4*S_thru.S11(i))/(S_thru.S21(i)); //T11/A

    //Equation THRU REVERSE...
    C6=(S_thru.S12(i))/(S_thru.S22(i)+C1); //T21/D
    ...

    //Equation Reflect
    X=(S_reflect.S22(i)+C2)/(S_reflect.S22(i)+C1);
    Y=(1+C3*S_reflect.S11(i))/(1+C4*S_reflect.S11(i));
    sol=sqrt((C5*X)/(Y*C6*C3)); //C

    A=1;B=C4;C=sol;D=C3*C;
    GAMMA_STD=(C*D*S_reflect.S11(i))/(A+B*S_reflect.S11(i));
    if ((real(GAMMA_STD)>0)&REFLECT_STD=="SHORT")||((real(GAMMA_STD)<0)&REFLECT_STD=="OPEN"), then,
        sol=sol*(-1);
    end;
```

Source Code : uW_TRL_calc.sci

```
.... A=1;B=C4;C=sol;D=C3+C;
.... 
.... K=[K; (sqrt(1/(A*D-B*C)))];
.... 
.... T_IN(6+1)=inv([A,B;C,D]);
.... T_OUT(6+1)=[C5,C2+C5;C6+D,C1+C6+D];
.... 
.... end;

//---- Reciprocity : K unsoving
..... x=phasmag(K(2:6))-phasmag(K(1:(6-1)));
=> a=(x<-90)*180+(x>90)*(-180);
=> a=conj(a,'r');
=> a=[:];
=> w=real(phasmag(K(1:6))+a);
.... cc=convj(S_thru.frequency/10^9,w*0+1)*w;
.... w=w-cc(2)*modulo(cc(2),180);
.... Kabs(K).*exp(%i*pi*w/180);

//---- BUILD-T-MATRIX
T11=[ ];T12=[ ];T21=[ ];T22=[ ];
for i=1:size(T_THRU.frequency,1),
.... T11=[T11;T_IN(i)(1,1)];T12=[T12;T_IN(i)(1,2)];T21=[T21;T_IN(i)(2,1)];T22=[T22;T_IN(i)(2,2)];
end;
Port_IN=clist(['T-parameters';'frequency';'T11';'T12';'T21';'T22'],T_THRU.frequency,(1./K).*T11,(1./K).*T21,(1./K).*T22);

T11=[ ];T12=[ ];T21=[ ];T22=[ ];
for i=1:size(T_THRU.frequency,1),
.... T11=[T11;T_OUT(i)(1,1)];T12=[T12;T_OUT(i)(1,2)];T21=[T21;T_OUT(i)(2,1)];T22=[T22;T_OUT(i)(2,2)];
end;
Port_OUT=clist(['T-parameters';'frequency';'T11';'T12';'T21';'T22'],T_THRU.frequency,K.*T11,K.*T12,K.*T21,K.*T22);

//---- Convert to S-param
S_IN=uW_T2S(Port_IN);
S_OUT=uW_T2S(Port_OUT);
S_R=uW_S2P_desembedding(S_reflect,S_IN,S_OUT);

varargout=list(S_IN,S_OUT,S_R);
endfunction
```